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## AN OUTLINE COURSE

IN

## MECHANICAL DRAWING



Instructor in Mechanical Drawing
IN
THE RHODE ISLAND SCHOOL OF DESIGN.

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## AN OUTLINE COURSE

## MECHANICAL DRAWING

FOR

EVENING DRAWING CLASSES,

BY

W. S. LOCKE,

Instructor in Mechanical Drawing

IN
THE RHODE ISLAND SCHOOL OF DESIGN.


PROVIDENCE, R. I.

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## INTRODUCTION.

This book is not a treatise on Mechanical Drawing, but an outline course on the subject. A mastery of its contents will not make the student an accomplished draughtsman, but it is hoped that such mastery, coupled with careful instruction, will help him a long way on toward that desired result.

The book is put out chiefly for that class of students who, from age or condition, are not able to take the time necessary to go through a complete course of study with the view of making themselves mechanical draughtsmen. The difficulty of ganging the needs of this body of students makes the success of any such effort problematical; nevertheless I make this trial with the hope of ultimate success.

> W. S. LOCKE.

Providence, R. I., Sept. 1888.

## FREE-HAND DRAWING.

The student, like the child, must "creep before he can walk," that is, do some preparatory work before he can stride along in his chosen path. In this preliminary work, the foundation, care should be taken. Study and work thoroughly, rather than fast. Make clear to your mind each subject as it is taken up, and by what is understood that which follows will be made clear.

Before writing became common, the saying, "The pen is mightier than the sword," was accepted by the world. In these days, when time is very valuable, drawings are generally used in all kinds of business.

Drawing is the universal language of man, and is mightier than the pen.

To make drawings or to understand them is becoming a necessity to all who work for a living, except the "day laborer."

At the beginning of the art we find free-hand drawing, that kind of drawing which is produced with the simple aids of pencil and paper. We must master something of this art before we can represent on paper what the " mind's eye" sees, be it Madonna or machine.

First, we must learn how to represent correctly that which we really see.

Figure 1 represents three views, top, end and side of a block. Figure 2 represents the same block as we naturally see it - not necessarily in this positionshowing three sides.

Once able to represent simple objects correctly in this way, and the student has grasped firmly a fundamental principle of perspective. There has to follow simply the training of the hand to do what the eye sees.


Our pictorial papers and magazines are proof that this kind of work " pays."

## geonetrical drawing.

Haring now begun to represent objects by drawings, we notice that a slight error in length or direction of a line injures the drawing of which it is a part.

We begin to study accuracy in these free-hand drawings and can easily see that some knowledge of Geometry, the science which treats of Position and Extension, must be a great and constant aid to the student of drawing. A great many treatises on Geometry hare been written, but none simple and
compact enough to be used in a course of drawing so limited as ours, consequently the principles of Geometry most necessary in the study of mechanical drawing are here condensed into thirty problems with useful definitions and conclusions. The problem before the student in each case is to construct a figure or drawing which shall graphically represent what the "solution" describes. This is practice in both geometry and drawing and a preparation for following work.

As the problems will require more aids than freehand drawing, it may be well here to say something about

## INSTRUMENTS.

A complete outfit for mechanical drawing may be bought for $\$ 15.00$, although half the money will buy instruments with which much may be done. It shonld comprise drawing board, T square, two triangles, two rubbers, two pencils, 12 in . steel scale, 1 pair $5 \frac{1}{2}$ in. compasses, 1 ruling pen, and 1 set (3) spring dividers. The compasses should have a fine needle point in one stock, and have the other stock fitted with pen and pencil points and a lengthening bar. The compasses are used in making circles and ares of circles. In using them, they should be used by the thumb and finger only. Never take hold of the points while making an arc.

In using the ruling pen, always be sure to hold it in a plane perpendicular to the face of the triangle.

Always nse the $T$ square with the left hand, and always work from the same side of it, for then your work will be regular.

Care must be taken all the time, for the art of making a good drawing comprehends the art of taking pains in details.

Having learned to take pains the habit is formed and unconsciously becomes easy.

## GEOMETRY.

## DEFINITIONS.

Definition. Geometry is the Science of Position and Extension.

Definition. A Point has merely position, without any extension.

Definition. Extension has three dimensions: Length, Breadth and Thickness.

Definition. A Line has only one dimension, namely, length. All lines are either straight or curved.

Definition. A Surface has two dimensions: Length and breadth.

Definition. A Solid has the three dimensions of extension ; length, breadth and thickness.

The Position of a Point is determined by its Direction and Distance from any known point.

Definition. A Plane is a surface in whic! any two points being taken, the straight line joining those points lies wholly in that surface.

Aciom. A straight line is the shortest way from oue point to another.

THE ANGLE.
Definitions. An Angle is formed by two lines meeting or crossing each other.

The Vertex of the angle is the point where its sides meet.

The magnitude of the angle depends solely upon the difference of direction of its sides at the vertex.

The magnitude of the angle does not depend upon the length of its sides.

When one straight line meets or crosses another, so as to make the two adjacent angles equal, each of these angles is called a Right angle, and the lines are said to be perpendicular to each other.

Thus the angles $A B C$ and $A B D$ (fig. 3) being equal, are right angles.

An Acute angle is one less than a right angle, as $K$ (fig. 3).

An Obtuse angle is one greater than a right angle, as $X$ (fig. 3).

Parallel Lines are straight lines which have the same Dircetion, as $A B, C D$ (fig. 3).

Parallel lines cannot meet, however far they are produced.

## POLYGONS.

Definitions. A plane figure is a plane terminated on all sides by lines.

If the lines are straight, the space which they contain is called a rectilineal figure, or polygon ( $P$ fig. 3).

The polygon of three sides is the most simple of these figures, and is called a triangle; that of fonr sides is called a quadrilateral; that of five sides, a pentagon; that of six, a hexagon, \&c.

A triangle is denominated equilateral ( $E$ fig. 3), when the three sides are equal, isosceles ( $I$ fig. 3 ), when two only of its sides are equal, and scalene ( $S$ fig. 3), when no two of its sides are equal.

A right-triangle is that which has a right angle. The side opposite to the right angle is called the hypothenuse. Thus $A B C$ (fig. 3) is a triangle rightangled at $A$, and the side $B C$ is the hypothennse.

Among quadrilateral figures, we distinguish;
The square (Sq. fig. 3), which has its sides equal, and its angles right-angles.

The rectangle ( $R$ fig. 3), which has its angles right angles, without having its sides equal.

The parallelogram ( $P$ fig. 3 ), which has its opposite sides parallel.

The rhombus or lozenge ( $P h$. fig. 3), which has its sides equal without haring its angles right angles.


The trapezoid (T fig. 3), which has two only of its sides parallel.

A diagonal is a line which joins the vertices of two angles not adjacent, as $A C$ in the figure of the parallelogram.

THE CIRCLE.
Definitions. The circumference of a circle is a curred line, all the points of which are equally distant from a point within, called the centre.

The circle is the space terminated by this curred line.

The radius of a circle is the straight line, as $A B$, $A C, A D$ (fig. 3), drawn from the centre to the circumference.

The diameter of a circle is a straight line, as $B D$, drawn through the centre, and terminated each way by the circumference.

A semicircumference is one half of the circumference, and a semicircle is one half of the circle itself.

An arc of a circle is any portion of its circumference, as BFE.

The chord of an are is the straight line, as $B E$, which joins its extremities.

The segment of a circle, is a part of a circle comprehended between an arc and its chord, as $E F B$.

A tangent (fig. 3) is a line, which has only one point in common with the circumference, as $H D$.

A polygon is said to be circumscribed abont a circle, when all its sides are tangents to the circumference; (Fig. 3) and in this case the circle is said to be inscribed in the polygon.

A polygon is inscribed in a circle when all its vertices are in the circumference of the circle, (fig. 3.)

## PROBLEMS

## Plain Geometry.

1. Problem. To find the position of a point in a plane, having given its distances from two known points in that plane.

Solution. Let the known points be $A$ and $B$. From the point $A$ as a centre, with a radius equal to the distance of the required point from $A$, describe an arc. Also, from the point $B$ as a centre, with a radius equal to the distance of the required point from $B$, describe an arc cutting the former arc ; and the point of intersection $C$ is the required point.
a. By the same process, another point $D$ may also be found which is at the given distances from $A$ and $B$, and either of these points therefore satisfies the conditions of the problem.
b. - If both the radii were taken of equal magnitudes, the points $C$ and $D$ thus found would be at equal distances from $A$ and $B$.
c. The problem is impossible, when the distance between the known points is greater than the sum of the given distances or less than their difference.
d. If the required point is to be at equal distances from the known point, its distance from either of them must be greater than half the distance between the known points.
2. Problem. To divide a given straight line $A B$ into two equal parts ; that is, to bisect it.

Solution. Find by $\S b$, a point $C$ at equal distances from the extremities $A$ and $B$ of the given line. Find also another point $D$, either above or below the line, at equal distances from $A$ and $B$. Through $C$ and $D$ draw the line $C D$, which bisects $A B$ at the point $E$.
3. Problem. At a given point $A$, in the line $B C$, to erect a perpendicular to this line.

Solution. Take the points $B$ and $C$ at equal distances from $A$; and find a point $D$ equally distant from $B$ and $C$. Join $A D$ and it is the perpendicular required.
4. Problem. From a given point $A$, above the straight line $B C$, to let fall a perpendicular upon this line.

Solution. From $A$ as a centre, with a radius sufficently great, describe an are cutting the line $B C$ in two points $B$ and $C$; find a point $D$ below $B C$, equally distant from $B$ and $C$, and the line $A D E$ is the perpendicular required.
5. Problem. To make an arc equal to a given are $A B$, the centre of which is at the given point $C$.

Solution. Draw the chord $A B$. From any point $D$ as a centre, with a radius equal to the given radius $C A$, describe the indefinite arc $F H$. From $F$ as a centre, with a radius equal to the chord $A B$, describe an arc cutting the arc $F H$ in $H$, and we have the arc $F H=A B$.
6. Problem. At a given point $A$, in the line $A B$, to make an angle equal to a given angle $K$.

Solution. From the vertex $K$, as a centre, with any radius describe an arc $I L$ meeting the sides of the
angle ; and from the point $A$ as a centre, by the preceding problem, make an are $B C$ equal to $I L$. Draw $A C$, and we have $A=K$.
7. Problem. To bisect a given are $A B$.

Solution. Find a point $D$ at equal distances from $A$ and $B$. Through the point $D$ and the centre $C$ draw the line $C D$, which bisects the arc $A B$ at $E$.
8. Problem. To bisect a given angle $A$.

Solution. From $A$ as a centre, with any radius, describe an are $B C$, and by the preceding problem, draw the line $A E$ to bisect the are $B C$, and it also bisects the angle $A$.
9. Problem. Through a given point $A$, above $B E C$, to draw a straight line parallel to a given straight line $B E C$.

Solution. Join $E A$, and, by problem 6, draw $A D$, making the angle $E A D=A E C$, and $A D$ is parallel to $B E C$.
10. Problem. Two angles of a triangle being given, to find the third.

Solution. Draw the line $A B C$. At any point $B$ draw the line $B D$, to make the angle $D B C$ equal to one of the given angles, and draw $B E$, to make $E B D$ equal to the other given angle, and $A B E$ is the required angle.
11. Problem. Two sizes of triangle and their included angle being given, to construct the triangle.

Solution. Make the angle $A$ equal to the given angle, take $A B$ and $A C$ equal to the given sides, join $B C$, and $A B C$ is the triangle required.
12. Problem. One side and two angles of a triangle being given, to construct the triangle.

Solution. If both the angles adjacent to the given side are not given, the third angle can be found by §10.

Then draw $A B$ equal to the given side, and draw $A C$ and $B C$, making the angles $A$ and $B$ equal to the angles adjacent to the given side, and $A B C$ is the triangle required.
13. Problem. The three sides of a triangle being given, to construct the triangle.

Solution. Draw $A B$ equal to one of the given sides, and, by $\S 1$, find the point $C$ at the given distances $A C$ and $B C$ from the point $C$, join $A C$ and $B C$, and $A B C$ is the triangle required.
a. The problem is impossible, when one of the given sides is greater than the sum of the other two.
14. Problem. To construct a right triangle, when a leg and the hypothenuse are given.

Solution. Draw $A B$ equal to the given leg. At $A$ erect the perpendicular $A C$, from $B$ as a centre, with a radius equal to the given hypothenuse, describe an arc cutting $A C$ at $C$. Join $B C$, and $A B C$ is the triangle required.
15. Problem. The adjacent sides of a parallelogram and their included angle being given, to construct the parallelogram.

Solution. Make the angle $A$ equal to the given angle, take $A B$ and $A C$ equal to the given sides, find the point $D$, by $\S 1$, at a distance from $B$ equal to $A C$, and at a distance from $C$ equal to $A B$. Join $B D$ and $D C$, and $A B C D$ is the parallelogram required.
a. If the given angle is a right angle, the figure is a rectangle; and, if the adjacent sides are also equal, the figure is a square.
16. Problem. To find the centre of a given circle or of a given are.

Solution. Take at pleasure three points $A, B, C$ on the given circumference or are; join the chords $A B$ and $B C$, and bisect them by the perpendiculars $D E$ and $F G$; the point $O$ in which these perpendiculars meet is the centre required.
17. Fiud by the same construction a circle, the circumference of which passes through three given points not in the same straight line.
18. Problem. Through a given point, to draw a tangent to a given circle.

Solution. If the given point $A$ is in the circumference, draw the radius $C A$, and through $A$ draw $A D$ perpendicular to $C A$, and $A D$ is the tangent required.
a. If the given point $A$ is without the circle, join it to the centre by the line $A C$; upon $A C$ as a diameter describe a circumference cutting the given circumference in $M$ and $N$; join $A M$ an $A N$, and they are the tancents required.
19. Problem. To inscribe a circle in a given triangle $A B C$.

Solution. Bisect the angles $A$ and $B$ by the lines $A O$ and $B O$, and their point of intersection $O$ is the centre of the required circle, and a perpendicular let fall from $O$ upon either side is its radius.
a. The three lines $A O, B O$, and $C O$, which bisect the three angles of a triangle, meet at the same point.
20. Problem. Upon a given straight line $A B$, to describe a segment capable of containing a given angle, that is, a segment such that each of the angles inscribed in it is equal to a given angle.

Solution. Draw $B F$, making the angle $A B F$ equal to the given angle. Draw $B O$ perpendicular to $B F$, and $O C$ perpendicular to the middle of $A B$. From $O$, the point of intersection of $O B$ and $O C$, with a radius $O B=O A$, describe a circumference.

If the angle $A B F$ is a right angle, the required segment is a semicircle. If angle $A B F$ is acute, the segment is more than a semicircle, and if angle $A B F$ is obtuse, the segment is less than a semicircle.
21. Problem. To divide a given straight line $A B$ into any number of equal parts.

Solution. Suppose the number of parts is, for example, six. Draw the indefinite line $A O$; take $A C$ of any convenient length, apply it six times to $A O$. Join $B$ and the last point of division $D$ by the line $B D$, draw $C E$ parallel to $D B$, and $A E$, being applied six times to $A B$, divides it into six equal parts.
22. Problem. To divide a given line $A B$ into parts proportional to any given lines, as $m, n, o$, etc.

Solution. Draw the indefinite line $A O$. Take

$$
A C=m, \quad C D=n, \quad D E=0, \text { etc. }
$$

Join $B$ to the last point $E$, and draw $C C^{\prime}, D D^{\prime}$, etc., parallel to $B E . \quad C^{\prime}, D^{\prime}$, etc., are the required points of division.
23. Problem. To find a mean proportional between two given lines.

Solution. Draw the straight line $A C B$. Take $A C$ equal to one of the given lines, and $B C$ equal to the other. Upon $A C B$ as a diameter describe the semicircle $A D B$. At $C$ erect the perpendicular $C D$, and $C D$ is the required mean proportional.
24. Prollem. To divide a given straight line $A C B$ at the point $C$ in extreme and mean ratio, that is, so that we may have the proportion ;

$$
A B: \quad A C=A C: C B
$$

Solution. At end $B$ erect the perpendieular $B D$ equal to half of $A C B$. Join $A D$, take $D E$ from $D$ on $A D$ equal to $B D$, and $A C$ equal to $A E$, and $C$ is the required point of division.

## REGULAR POLYGONS.

Definitions. A regular polygon is one which is at the same time equiangular and equilateral.

Hence the equilateral triangle is the regular polygon of three sides, and the square the one of four.

An equilateral polygon is one which has all its sides equal; an equiangular polygon is one which has all its angles equal.
25. Problem. To inscribe a square in a given circle.

Solution. Draw two diameters $A B$ and $C D$ perpendicular to each other ; join $A D, D B, D C, C A$; and $A D B C$ is the required square.
26. Problem. To inscribe in a given circle a regular hexagon.

Solution. Take the side $B C$ of the hexagon equal to the radius $A C$ of the circle, and, by applying it six times round the circumference, the required hexagon $B C D E F G$ is obtained.
27. Problem. To inscribe a regular decagon in any circle.

Solution. Divide the radius of a (three-inch) circle in extreme and mean ratio. (Problem 24.)

The longer part is equal to one side of the regular decagon required. Apply it ten times to the circumference, and join the points by straight lines, making the decagon.

Make a pentagon by joining the alternate vertices of the decagon.
28. Problem. To circumscribe a circle about a given regular polygon $A B C D$, \&c.

Solution. Find, by Problem 17, the circumference of a circle which passes through three vertices $A$, $B, C$; and this circle is circumscribed about the given polygon.
29. Problem. To inscribe a circle in a given regular polygon $A B C D$, \&c.

Bisect two sides of the polygon by perpendiculars, the point of intersection is the centre of the required circle.

The sides of the polygon become tangents to the circle.

AREAS.
Definitions. Equivalent figures are those which have the same surface.

The area of a figure is the measure of its surface.
The unit of surface is the square whose side is a linear unit; such as a square inch or a square foot.

The area of a square is the square of one of its sides.

A parallelogram is equivalent to a rectangle of the same base and altitude.

The area of a parallelogram is the product of its base by its altitude.

Parallelograms of the same base are to each other as their altitudes; and those of the same altitude are to each other as their bases.

All triangles of the same base and altitude are equivalent.

The area of a triangle is half the product of its base by its altitude.

Every triangle is half of a parallelogram of the same base and altitude.

The area of a trapezoid is half the product of its altitude by the sum of its parallel sides.
30. Problem. To make a square equivalent to the sum of two given squares.

Solution. Construct a right angle $C$ (see fig. 3) ; take $C A$ equal to a side of one of the given squares ; take $C B$ equal to a side of the other; join $A B$, and $A B$ is a side of the square sought.

A square may be found equivalent to a given triangle, by taking for its side a mean proportional between the base and half the altitude of the triangle.

A square may be found equivalent to a given circle, by taking for its side a mean proportional between the radius and half the circumference of the circle.

## Orthographic Projections.

All mechanical drawing is founded on mathematics -principally on Plane, Solid and Descriptive Geometry.

Drawings are so made as to present to the eye, situated at a particular point, the same appearance as the magnitude or object itself, were it placed in the proper position. The representations thus made are the projections of the object.

The planes upon which these projections are usually made are the planes of projection. The point at which the eye is situated is the point of sight.

Definition. When the point of sight is in a perpendicular, drawn to the plane of projection, through any point of the drawing, and at an infinite distance from this plane, the projections are Orthographic.


Definition. When the point of sight is within a finite distance of the drawing, the projections are Scenographic, commonly called the Perspective.

In Orthographic projections, three planes of projection (sometimes two suffice) are used, at right
angles to each other, one horizontal and the other two vertical, called respectively the horizontal and vertical planes of projection, and denoted by the letters $H$ and V.

Let a plane rectangular cross (fig. 5) be imagined self-suspended near a lower corner of a room, or between three sheets of paper, placed in similar position, namely : at right angles to each other ; the three principal dimensions, length, breadth and thickness, of the cross being each perpendicular to one of the sheets of paper-which serve as the three planes of projection. As indicated by the dotted lines, let perpendiculars be drawn from the principal points of the cross to each plane of projection.


Let the two vertical sheets be now laid down on a table, keeping the top of the cross in line on both. Now (fig. 4) we have the three projections of the cross on one plane in the manner in which it is proper to represent them as Orthographic projections.

It is easily seen that neither of these projections is a correct representation of the cross as we see it, and also, that collectively the three projections truly represent the cross in length, breadth and thickness. Here then is the value of this method of representing objects.

All that these projections need to make them working drawings, are the dimensions in figures. The projection on $H$ is called the Plan, and the two on $V$ and $P$, are called Elevations.

The representation in (fig. 5) is in Perspective. Figures 1 and 2 represent the whole principle in the same manner.

## GROUND LINE.

It is to be observed that the line which (in fig. 5) is made by the intersection of $H$ and $V$, is prescrred in fig. 4. It is called the Ground Line. The representations of the object above the Ground Line are called Elevations, and the one below is called the Plan, of the object.


Returning now to fig. 5 , it will be seen that the Plan is drawn upon the plane you look down upon, and the elevations upon planes you look upon horizontally.

After a little experience, the Ground Line becomes as imaginary as the Equator, but like the latter serves its purpose.

Fig. 6 represents in Plan and Elevation a triangular prism, fig. 7 a rectangular prism, fig. 8 a square pyramid, fig. 9 a hexagonal pyramid, fig. 10 a right cylinder, and fig. 11 a cone.

These names, objects and representations should be kept in mind, for they will be referred to many times.

So far we have confined ourselves to projections of objects placed at right angles to the planes of projection, but it will be easily understood that in making drawings of machines or houses, we shall find many lines which are not so related to natural planes of projection already described. For an example, let us take the rectangular prism (fig. 7) just used. To keep us in the right way, we figure the corners of the front side (fig. 12). In the plan the figures double, but make no confusion so long as we have the Elevation to look at.

Tip the prism now, so that the base line 3,4 will make an angle of $30^{\circ}$ with the ground line, keeping the plane of the face $1,2,3,4$, parallel to the ground line.

To make a plan of the prism in this new position: As the different representations or views of an object are supposed always to be in positions perpendicular to each other, the corner 1 , for example, will be found in a perpendicular to ground line. As the prism was not inclined to the vertical. plane, the desired comer will be found in a line through 1 of the plan, parallel to the ground line.

The perpendicular from 1 in the elevation, and the parallel from 1 in the plan meet, making 1 of the new plan.

In the same way seven other coruers are found and the new plan finished.

We have now two views of the prism as it is inclined to $H$. Move this new plan to the right and incline it to $V$ at an angle of $45^{\circ}$. Now from the third plan draw a perpendicular, and from the second elevation draw a parallel from corner 1 . The point in which these two lines meet is corner 1 in the third elevation. One by one seven other points may be found, completing the elevation of the prism as it appears inclined to both planes of projection.

This work brings us to a point where we may attempt the problem known as: "To find the true length of a line."

In the plan of fig. 9 it is clear that the lines $B E$ and $C E$ forming two sides of one triangular face of the pyramid, are not seen in their true length. $A E$ and $E D$ are projected in their full lengths on $V$, becailse their projections on $H$ are parallel to $G L$.

By the plan it will be seen that $E$ is equally distant from $A B C$ and $D$.

Turn the prism on its perpendicular axis until $C$ occupies position now occupied by $D . E C$ now being parallel to the ground line will be projected on $V$ in its true length.

Note: $E$ is supposed to be at the apex of the pyranid, with $A, B, C$ and $D$ at the comers beginning with $A$ at the extreme left, $B$ and $C$ at the bottom, and $D$ at the right.

Suppose now a more difficult problem: That we were required to find the length of the line 1,4 in the third elevation of the rectangular prism (fig. 12).

First, revolve the prism into the second fosition (of fig. 12) with the plan of 1,4 parallel to $G L$. From 1 and 4 of this second position draw perpendiculars to $G L$, and from 1 and 4 of the third elevation draw parallels to $G L$; their intersections will be at the ends of 1,4 of the line in its true length.

## CONIC SECTIONS.

The conic sections are so called because they are sections of a cone.

We have had a definition of a plane. Imagine two such surfaces passed through a solid, at a distance from each other of less than the thousandth part of an inch. The slice of the solid between the planes is termed a Section. The Conic Sections are taken from a right cone and are, the Triangle, Circle, Ellipse, Parabola, and Hyperbola. The Triangle is a section cut from a cone by two planes passed throngh the apex perpendicular to the base.

A Circle is a section of a right cone cut at right angles to the axis.

The Ellipse is a curved section cut at any angle to the axis, large enough to cut both sides of the cone.

The Hyperbola is a curved section cut from the cone parallel to the axis and perpendicular to the base.

The Parabola is a curved section cut from a right cone parallel to one of the sides as itappears in elevation.

## InTERSECTIONS AND DEVELOPMENTS.

As the memory will easily recall, there are many lines seen on a manufactured article, or on a drawing of it, that are not lines of any individual part of the article, but lines that oceur where two or more forms intersect or join each other. Such lines are called intersection lines.

Intersections is the name given to that part of geometrical drawing that treats of the intersection lines and their correct delineation.

It will be seen at once that a thorongh knowledge of geometrical solids will be necessary to the student
who desires to take a full course in intersections. On the other hand, we all have a fair understanding of many geometrical forms that meet our eyes in combination every day, and with these we will make a beginning.

We will suppose a triangular prism passed through a right cylinder (fig. 13).

First make a plan and two elevations of the cylinder. In the middle of the elevation which is projected from the plan, draw the end elevation of the prism.


Now draw the plan assuming that the prism projects from the cylinder at either side, and that centre line of prism and cylinder coincide in $D, E, C$.

In the side elevation it is evident that there will be an intersection line ; that none appears in the plan and end elevation is evident becanse in the plan it coincides with the outline of the cylinder, and in the end elevation it coincides with the outline of the prism.

We can lay out the top and bottom lines, and ends on the side elevation from the other views. The point where the lower line of the prism pierces the cylinder is found as follows: In the plan draw the line $Z, N$,
perpendicular to the diameter $Z, O$, through the point wherc the prism pierces the cylinder.

Lay off the distance $A, O$ from X to $Y$, the latter being the desired point.

The top point of the intersection line is on the circumference. To find other points in the intersection line, pass the planes 1 and 2 and proceed in the same way as in the case of the bottom line.

It is now required to develop the surface of the cylinder. In mechanical drawing this means to draw an equivalent plane figure. This may be illustrated by fitting the cylindrical surface with a covering of paper. When this paper is morolled and spread on a table, we have a surface equivalent to that of the cylinder. It is now required to outline, in this development of the cylinder, the hole that the prism makes. Let $A, H$ (fig. 13) represent part of the development of the cylinder. Let a perpendicular at $D$, represent the axis of the prism. Lay off the arc $L$, $I I$ developed as a straight line on each side of the axis, making the line $R, P$. The third corner is found on the axis at the altitude of the prism from $R, P$. Intermerliate points are laid off from the axis on traces of the planes 1 and 2 in the same manner.

Any boiler maker will understand the value of these operations. To be able to lay out on a sheet of iron the size and shape of hole necessary to fit on a pipe or manhole after the boiler is made, will sometimes help a man a good step toward success.

## Isometric Projections.

An isometrical drawing is one that shows three sides of a solid body in one view.

Let three straight lines be drawn intersecting in a common point and perpendicular to each other, two of them being horizontal and the third vertical ; like the three adjacent edges of a cube.

Then let a fourth straight line be drawn through the same point, making equal angles with the first three, as the diagonal of a cube. If now a plane be passed perpendicular to this fourth line, and the straight lines and other objects be orthographically projected upon it, the projections are called Isometric.

The three straight lines first drawn are the co-ordinate axes; and the planes of these, taken two and two, are the co-ordinate planes. The common point is the origin. The fourth line is the Isometric Axis.

Since the co-ordinate axes make equal angles with each other, and with the plane of projection, it is evident that their projections will make equal angles with each other, two and two, that is, angles of $120^{\circ}$. Hence, fig. 14, if any three straiglit lines, as $A x, A y$ and $A z$, be drawn through the point $A$, making with each other angles of $120^{\circ}$, these may be taken as the projections of the co-ordinate axes, and are the directrices of the drawing.

It is further evident, that if any equal distances be taken on the co-ordinate axes, or on lines parallel to either of them, their projections will be equal to each other, since each projection will be equal to the distance itself into the cosine of the angle of inclination of the axes with the plane of projection.

The angle which the diagonal of a cube makes with either adjacent edge is known to be $54^{\circ} 44^{\prime}$; therefore the angle which either edge, or either of the co-ordinate axes, makes with the plane of projection will be the complement of this angle, viz., $35^{\circ} 16^{\prime}$.

If a scale of equal parts be constructed, the unit of the scale, being the projection of any definite part of either co-ordinate axis, as one inch, or one foot, will be one inch multiplied by the natural cosine of $35^{\circ}$ 16 . We may from this scale determine the true length of the isometric projection of any given portion of either of the co-ordinate axes, or of lines parallel to them, by taking from the scale the same number of units as the number of inches or feet in the given distance. Conversely, the true length of any line in space may be found by applying its projection to the Isometric scale, and taking the same number of inches or feet, as the number of parts covered on the scale.

Or: The Isometrical length of a line, is the true length of a line multiplied by the natural cosine of $35^{\circ} 16^{\prime}$. Now this cosine is .816 ; hence, if we multiply the true length of a line-say one inch long-by .816, we will get the isometrical length of the line, that is, one iuch multiplied by .816 , which equals .816 (thousandths) of an inch, or about ( $\frac{13}{16}$ ) thirteen sixteenths of an inch.

Now, if we have a $35^{\circ} 16^{\prime}$ triangle, and a scale thirteen sixteenths full size, we have the special tools necessary to make an isometrical drawing.

A much easier way is generally adopted.
It is customary to use a $30^{\circ}$ triangle in place of a $35^{\circ} 16^{\prime}$ triangle, and a full size scale in place of a $\frac{13}{16}$ scale.

Since in most of the frame work connected with machinery, and in varions kinds of buildings, the principal lines to be represented, occupy a position similar to the co-ordinate axes, namely, perpendicular
to each other, one system being vertical, and two others horizontal, the isometric projection is used to great advantage in their representation. A still greater advantage arises from the fact that in a drawing thus made, all lines parallel to the directrices are constructed on a full size scale.

If the isometrical projection of a point be required, the following operation is sufficient :



Thus in fig. 14, let $A Z, A Y$ and $A X$ be the directrices, $A$ being the projection of the origin. On $A$, $Y$, lay off $A, P$ equal to the distance of the point from the co-ordinate plane $Y, Z$.

Through $P$ draw $P, M I^{\prime}$ parallel to $A, Y$ and make it equal to the distance of the point from the plane XZ.

Throngh $\mathrm{MI}^{\prime}$, draw $\mathrm{MI}^{\prime}$, M parallel to $A X$ and make it equal to the third given distance, and MI will be the required projection.

The projection of any straight line parallel to either of the co-ordinate axes may be constructed by finding, as above, the projection of one of its points, and drawing through this, a line parallel to the proper directrix.

If the line is parallel to neither of the axes, the projections of its ends may be found, as above, and joined by a straight line, which will be the projection required.

The projections of curves may be constructed by finding a sufficient number of the projections of their points.

## PROBLEM.

To construct the isometric projection of a cube. Let the origin be taken at one of the upper corners of the cube, the base being horizontal, and let $A X, A Y$ and $A Z$ (fig. 14) be the directrices.

From $A$, on the directrices, lay off the distances $A X, A Z$ and $A Y$, each equal to the length of the edge of the cube.

These lines will be the projections of the three edges of the cube which intersect at $A$. Through $X, Y$ and $Z$ draw $X e, X g, Y e, Y c, \not / c$ and $Z g$, parallel to the directrices, completing the three equal rhombuses $A, X, e, y$, etc.

These will be the projections of the three faces of the cube-which are seen-and the representation will be complete.

## Working Drawings.

It is the opinion of the author that a fair understanding of the principles touched upon in previous chapters is necessary to enable an ordinary draughtsman to do his work ; and conversely, having such understanding he will be able to make good working drawings. A working drawing is a drawing made for workmen to use in making the thing drawn. A working drawing sloould represent clearly the form, material and dimensions of the thing to be constructed. The design must be made perfectly correct, because from it are taken the dimensions for the working drawing. Dimensions should be taken several times before being put on a drawing and the workman required to work from the dimensions; this requirement is necessary because the paper on which a drawing is made will shrink and swell with the changes in the weather, making it mntrustworthy as a guide to correct figures. It will be noted from what has been said that the most important part of a working drawing is the dimensions.

A sheet of working drawings should always bear a a title-if possible in the lower right hand corner. The title should answer with utmost brevity the following questions: What is it? What scale? When and who by? Some manufacturers require, in addition, the name of the firm and the number of sheets of working drawings to complete the machine.

Figure 15 is a working drawing for a small jourual or box. It will be noticed that all necessary figures are given, and no more except in one case. It is best to make as few figures as possible consistent with
clearness. Figures should not disfigure the outlines of the drawing, and should be placed as nearly as possible in the place in which they would naturally be found. It would be easy to write many pages of sound instruction in the art of making working draw-

ings, and the matter would be almost as easily forgotten. It is better to fasten the necessary principles and manipulations in the memory by actual practice in making drawings, with the help of a teacher.


## DESIGN.

Having learned to make working drawings, there is a natural desire to take the next step, and learn to design the buildings or machines that require the explanatory working drawing. The designing of cams and gearing will be explained, and a finished design is shown of a machine designed to roll hot bar steel into finished forgings for the market. While it is not the custom to shade such designs, the present one is so finished to give the student some instruction in that art.

In the subjects just mentioned much will be learned, incidentally, of designs, in a class of facts that soon become common-place to the draughtsman or designer, but are as necessary to him as his paper.

A good working table of strength of materials is given, but it is beyond the scope of this work to teach a subject that requires original thought and oftentimes original research.



## CAMS.

The subject of cams is taken up here because it is easy of mastery, convenient of application and economical in use.

Definition: (Worcester.) "Cam Wheel-A wheel formed so as to move eccentrically, and produce a reciprocating and interrupted motion in some other part of machinery comnected with it."

This is rather misty for a definition, but is as good as may be found outside regular mechanical writing.

On the cam sheet, fig. 1 shows the simplest form of cam, sometimes called a wiper. The part moved by it is kept in contact with it by a spring or by its own weight. The outline is generally similar to that of fig. 1. The width of face is proportioned to the work to be done. 'This form of cam is often used for trip hammers. Fig. 2 shows a path cam. A "ball," as it is called, being in fact a right cylinder, moves in the curved path as the cam wheel revolves, and gives any points of the comnected arms or link work, a positive and regularly irregular motion. This last phrase, " regularly irregular motion," is used in deference to the common expression used in speaking of revolutionary movements.

In the popular acception of the term, such movements are " regular'" when in circles or arcs of circles, and "irregular" otherwise. You will learn, however, that cam movements may be as regular as crank movements - indeed, a crank cam is not difficult of design or construction.

To return now to the cam represented by fig. 2.
The cam roll in this case is fixed at the end of one arm of a bell crank. As the cam revolves, the crank swings through an are about its axis $X$. By changing the proportionate length of the arms of the bell crank, the motion of the knife $T$ may be made proportionately faster or slower than that of the roll. On the chart, the length of carn lever is taken to be equal to the distance from the point of support $X$, of the bell crank to the centre of the cam.

As the roll travels in or out from the centre of the cam, it always moves in an arc of a circle of this radius.

If the point of support be changed, this radius is not, in platting the position of the roll in its successive stages. Shortening $L X$, quickens the movement of $T$, but decreases its power. Lengthening $L X$, makes $T$ move slower, increasing its power. The whole arrangement is similar to that used in paper folding machines.

To design the cam, we must know first, direction, amount and velocity of movement reguired of $T$, then points $X$ and $Y$ are selected - and this part of the work is generally very tronblesome - after which the size and "throws" of the cam must be so made as to accomplish the work. Sometimes the space available for a cam is much smaller than the movement required from it, in which case the throws are timed correctly and the rest of the problem solved in the link work. In the chart it is assumed that the forward stroke or work of the cam must be accomplisherl in one-eleventh of a revolution of the cam, after which $T$ is returned easily to its position of rest of one-half revolution.

Much care must be taken in turning the corners between the radial arcs 15 and 20 . If a too quick turn is attempted the roll will bind. The circles in the path of the cam roll represent some of its positions.

Where the change of direction of the path is rapid, these positions must be near each other. If each one of these circles shows a part of its circumference beyond all others on the inside line of the path of the cam, the curre in the path is possible - that is, practicable.

There are, then, but two indispensable principles to be kept in mind while laying out the path of a cam of this kind.

1 st. That during the change of direction of motion, the successive positions of the cam roll must be accurately platted.

2d. That each of these successive positions must form an integral part of the path. These two laws apply also to frog cams, for if that part, of this path cam, which is outside the inside line of the path were cut away, the remainder would be a frog cam.

Figures 3 and 4 are side and section views of a "square" cam with its immediate attachment, the " yoke." This cam is often nsed in sewing machines and boot and shoe machinery.

In the position shown, the upper and lower quadrants are ares of circles, having the axis of the cam for a centre, and are, in cam langnage, "rests." This is an unfortmate term here, for this cam has no rest, nor gives any to its attachments.

The secrets of drawing of this cam are but two:
1st. There is a point on each of the diagonals $E F, I H$, which is a centre for two ares of circles, each forming a part of the required cam ontline.

2d. The snm of the radii of these two ares is equal to the sum of the radii of the two ares of " rest."

Simple accuracy of drawing is all that is recquired, in addition to above information to prorluce this cam.

The eam $K$, on the chart, will, by means of its yoke pivoted at $N$, describe the "square" figure $H^{\prime} E^{\prime} I^{\prime} F^{\prime}$. This sort of motion is made exceedingly
simple, by means of this "square" cam, but is often accomplished by expensive link work.

Figures 5, 6 and 7 show an edge cam and methods used in its design. Fig. 5 shows it in comnection with roll and lever. Fig. 6 shows the ordinary division of the circumference into 8 divisions of $45^{\circ}$ each. Fig. 7 shows the development of the circumference with the path traced upon it. The largest diameter of the roll is used in tracing the path. It is assumed that three throws and three rests are required.

The first quarter, 1 to 3 , is a rest in the middle of the lever angle. In the next, 8th revolution, (3 to 4), the lever takes its extreme "back" position. The next quarter, 4 to 6 , is a rest in this extreme position. In the next quarter, 6 to 8 , the work is supposed to be executed.

The whole throw forward is effected at an easy angle in one quarter revolution. One sixty-fourth rest is given to hold the work, and seven sixty-fourths revolutions returns the lever to its first position.

It will be noticed that the cam roll for this cam is conical. This is necessary, because that part of the roll nearest $A$ must travel a much greater distance than the end nearest the centre of the cam. Now that your attention is called to the fact, you will readily comprehend this necessity ; but there are many cams of this kind running with rolls that are right cylinders simply because their designers forgot this simple fact. In fig. 5 the cam lever is shown at point 1 , with the centre line of the lever passing through the centre line of the throw of the cam. From 3 to 4 the roll drops seven-sixteenths of an inch, and from 4 to 6 the lever makes an angle with the centre line of the cam, as is shown by the dotted line drawn from $A^{\prime}$ to the correct position of the roll.

In laying ont the curves of the cam path at the ends of the "throws," much care must be taken not

to have sharp angles. The centre line is laid out that is - the centre line of movement. The changing of this line from a rest to a throw, or the reverse, is always made in the are of a circle. The radius of this arc must be at least one-eighth of an inch larger than the largest radins of the cam roll. This ensures a round comer for the roll. It is better, where possible, to make this arc one-quarter of an inch larger than the radins of the roll. The change in direction of this centre line can never be over $45^{\circ}$ for forward or working throws. $30^{\circ}$ angles should not be exceeded.

With these descriptions of the work on the chart, and with the chart before you, your foundation is laid for an easy mastery of all the principles involved.

When cams are to be cut, it is best to make the for-mer- called the "leater" in this case - much larger than the cam, so as to aroid inaccuracies and unsteady lines in the cam. The cam blank is also marked. This is accomplished by duplicating the development on a piece of thin tin, which is then fastened to the blank and the ontline prick-punched through. Sometimes the development is on strong manilla paper, which is wrapped on the cam and pricked through.

Edge cams are generally, and path cams always, cast with the path in them. Face cam patterns and castings are sometimes made so nicely that a coldclisel aud file will in a very few minutes smooth the casting sufficient for the roll. Such cams wear a long time.

Usually cams are cut from leaders of double their size. These leaders are disks of cast iron, half an inch thick, turned smoothly and emery-polished, so that the scriber lines may be seen easily. The leader for such a cam as No. 5 , for example, would be a plain disk eight inches in diameter with the throw of seren-eighths of an inch and two throws of sevensixteenths of an inch laid off on its edge. This, when
ready for work, would not bear the slightest resemblance to the cam-except that its periphery would be a transcript of the cam's path. From this description the common method of laying ont cam leaders is easily understood.

## GEARING.

In this chapter the student may find 1st, a clear indication of methods of drawing three kinds of gear teeth. 2d, a diagram and its description by which gear wheels may be proportioned ; and 3d, a few practical hints. Several standard works have been consulted, especially the treatise published by the Brown \& Sharpe Mfg. Co., from which are taken, by permission, methods for delineating single-curve and involute forms of teeth, and also useful formulas and explanations. While a larger portion of the space is used in describing methods used in drawing teeth than in explanation of the designing of gears, the student should direct his chief thought to the latter work, since there are several manufacturers who make better gear teeth than any inexperienced man can hope to.

Let us imagine two cylinders, mounted on parallel axes, having their convex surfaces in contact. If now we turn one cylinder, the adhesion of its surface to the surface of the other will make that turn also. The surfaces touching each other, if there is no slip, will evidently move through the same distance in a given time. This surface speed is called linear velocity. Linear velocity is the distance a point moves in a given direction in a given time. These cylinders in turning abont their axes also pass throngh angles whose verti-
ces are at the axes of the cylinders. The angular distance passed through in a given time is called angular velocity. If one cylinder is twice as large as the other, the smaller will make two turns while the larger makes one, but the linear velocity of the cylinders is equal.

This combination would be a very useful one in mechanism if we could be sure that the cylinders would not slip on each other.

Let us inagine grooves cut on the circumferences of the cylinders of a size equal to the spaces between the grooves, and the material taken out in making the groove placed on the spaces between the grooves. The spaces are called lands, and the parts placed upoin them addenda. A land and its addendum is called a tooth. A toothed cylinder is called a gear. Two or more gears with teeth interlocking are called a train. The circumference of the cylinders, without teeth, is called the pitch circle. This circle exists geometrically in every gear and is called the pitch circle, or the primitive circle. In the study of gear wheels it is the problem to so shape the teeth that the pitch circles will just roll on each other without slipping.

The groove between two teeth is called a space. In cut gears the width of space and thickness of tooth at pitch line are equal.

The circular pitch is the distance measured on the pitch line, or pitch circle, which embraces a tooth and a space. In cast gears the tooth is from .46 to .48 of the circular pitch.

If we conceive the pitch of a pair of gears to be the smallest possible, we finally reduce the teeth to mere lines on the original pitch surfaces. These lines are called elements of the teeth. Gears may be classified with relation to the elements of their teeth, and also with relation to the direction of their shafts.

## CLASSIFICATION.

First.-Spur Gears ; those gears connecting parallel shafts and whose tooth elements are straight.

Second.-Bevel Gears ; those gears connecting shafts whose axes meet when sufficiently prolonged, and the elements of whose teeth are straight lines. In berel gears the surfaces that touch each other, without slipping, are upon cones or parts of cones, whose apexes are at the point where the centre of their shafts meet.

Third.-Worm Gears ; those whose axes meither meet nor are parallel, and the elements of whose teeth are helical, or screw-like. A modification of this form of tooth is the skew-bevel wheel, which is used in some cases with a smooth surface which is a zone or frustrum of an hyperboloid of revolution. A hyperboloid of revolution is a surface resembling a dicebox, generated by the revolution of a straight line around an axis from which it is at a constant distance, and to which it is inclined at a constant angle. Of modifications of the spur-gear we have the internal gear, the elliptical gear, the segment and the rack.

There are almost endless modifications of the different classes of gears, and anything like a careful description of their uses, and the methods of designing them, would make a book of one hundred pages octavo size, and consequently far beyond our present range. We will consider Bevel Gears and Spur Gears. We will use the following abbreviations :

Let $D=$ the diameter of addendum, or full size circle.

Let $D^{\prime}=$ the diameter of pitch circle.
Let $P^{\prime}=$ the circular pitch.
Let $t=$ the thickness of tooth at pitch line.
Let $s=$ the addendum, or face of tooth.
Let $f=$ the clearance.
Let $D^{\prime \prime}=2 s$, or working depth of toath.

Let $D^{\prime \prime}+f=$ whole depth of space.
Let $N=$ number of teeth in one gear.
Let $n=3.1416$, or circumference when diameter is 1 .

If we multiply the diameter of any circle by $n$, the product is the circumference of that circle. If we divide the circmmference by $n$, the quotient will be the diameter of that circle.

The circular pitch and number of teeth in a wheel being given, the diameter of the wheel and size of tooth parts are found as follows:

Dividing by $3.1416=$ multiplying by $\frac{1}{31+16}=$ . 3183 ; hence, multiply the circumference of a circle by .3183 and the product is the diameter of the circle.

Multiply the circular pitch by .3183 and the product will be the same part of the diameter of the pitch circle that the circular pitch is of the circumference of pitch circle. This part, or modulus, is called a diameter pitch.

The diameter pitch $=$ addendum of tooth $=s$. Circular pitch multiplied by $.3183=s$, or $.3183 P^{\prime}$ $=s$.

The number of teeth in a wheel multiplied by a diameter pitch equals diameter of pitch circle, $N s=$ $D^{\prime}$. Add two to the number of teeth, multiply the sum by $s$ and the product will be the whole diameter, $(N+2) s=D$. One-tenth of thickness of tooth at pitch line, equals amount added to bottom of space for clearance, $\frac{i}{10}=f . \quad \frac{N_{8}}{2}=\frac{n^{2}}{2}=$ Radius of pitch circle.

Distance between centres of two wheels equals the sum of the two pitch circle radii.

In making drawings of gears and in cutting racks, it is necessary to know the circular pitch in whole inches and the natural divisions of an inch, as onehalf inch pitch, one-quarter inch pitch, etc., but since it is difficult to measure the circomference of the pitch
circle and divide it into equal parts, it is much better that the diameter of a gear should be of a size conveniently measured.

The same applies to the distance between centres. Hence it is generally more convenient to assume the pitch in terms of the diameter. A definition of a diameter pitch and the method of obtaining it from the circular pitch has been given.

If the circumference of the pitch circle is divided by the number of the teeth in the gear, the quotient will be the circular pitch. If the diameter of the pitch circle is divided by the number of the teeth, the quotient will be a diameter pitch. Thus, if a gear has forty-eight teeth and a pitch diameter of twelve inches, the diameter pitch is twelve inches divided by fortyeight, or one-quarter of an inch, Naturally, in deciding dimensions of teeth for a gear, a diameter pitch of some convenient part of an inch is taken.

In speaking of diameter pitch, only the denominator of the fraction is named. One-third of an inch diameter pitch is called 3 diametrical pitch. Diametrical pitch is the number of teeth to one inch of diameter of pitch circle. Represent this by $P$. Thus, onequarter inch diameter pitch becomes 4 diametrical piteh, or $4 P$, becanse there would be four teetl on the gear to every inch of diameter of its pitch circle.

The circular pitch and different parts of the teeth are derived from the diametrical pitch as follows:
(1) $\frac{3.1416}{P}=P^{\prime}$, or 3.1416 divided by the diametrical pitch is equal to the circular pitch.
(2) $\frac{1}{6}=P$, or one inch divided by the thickness of one tooth equals number of teeth to one inch.
(3) $\frac{1.57}{\bar{p}}=\ell$, or $1 . \overline{5} 7$ divided by the diametrical pitch gives thickness of tooth at pitch line.
(4) $\frac{N}{P}=D^{\prime}$, or number of tecth in a gear divided by the diametrical pitch equals diameter of the pitch circle. The diameter of the pitch circle of a wheel
having 60 teeth, $12 P$, would be, consequently, five inches.
(5) $\frac{N \text { plus } 2^{2}}{P}=D$, or, add 2 to the number of teeth in a wheel and divide the sum by the diametrical pitch, and the quotient will be the whole diameter of the gear or the diameter of the addendum circle. The diameter of gear blank for a gear of sixty teeth, $12 P$, would be, consequently, $5_{\overline{1}} \frac{2}{2}$ inches.
(6) $\frac{N}{\bar{D}}=P$, or number of teeth divided by diameter of pitch circle, in inches, gives the diametrical pitch.
(7) $\frac{N \text { plus } 2}{D}=P$, or add 2 to the number of teeth, divide by whole diameter and quotient will be diametrical pitch. $P D^{\prime}=N$, or pitch circle diameter multiplied by diametrical pitch equals number of teeth in the gear.

The reverse of formula (1) is true, (8) $\frac{3.1416}{P^{\prime}}=P$.

## Single Curve Gears,

Siugle curve teeth are so called becanse their working surfaces have but one curve, which forms both face and flank of tooth sides. This curve is, approximately, an involute. In a gear of 30 tecth or more, this curve may be the single arc of a circle, whose radius is one-fourth the radius of the pitch circle. A fillet is added at the bottom of the tooth, to make it stronger, equal in radius to one-sixth the widest part of tooth space.

A cutter made to leave this fillet has the advantage of wearing longer than it would if brought up to a corner.

In gears of this class of less than 30 teeth, this fillet is made the same as just given, and sides of teeth formed of more than one arc, as will be shown later.

Having calculated the data of a gear of 30 teeth, we may proceed as follows :

1st. Draw pitch circle and point it off into parts equal to $\frac{1}{2}$ circular pitch.

2d. From one of these points, as $c$, (in sketch of " single curve bevel gear ") draw radius to pitch circle, and on this radius describe a semicircle; the diameter of this semicircle being equal to radius of pitch circle. Draw addendum, working depth, and whole depth circles.

3d. From the point $c$, where pitch circle, semicircle and outer end of radius to pitch circle meet, lay off a distance on semicircle equal to the fourth part of the radius of pitch circle, shown in the figure at $c y$. This is laid off as a chord.

4th. Through this new point $y$, upon the semicircle, draw a circle concentric to pitch circle. This last is called the base circle, and is the one for tooth ares. In the best practice, the diameter of this circle is made about forty-nine fiftieths of the diameter of pitch circle.

With dividers set to one-quarter of the radius of fitch circle, draw arcs forming sides of teeth, placing one leg of the dividers in the base circle and letting the other leg describe an are through a point in the pitch circle that was made in laying off the parts equal to one-half the circular pitch.

With dividers set to one-sixth widest part of tootli space, draw fillets for strengthening teeth at their roots. These fillet arcs should just touch the whole depth circle and the sides of teeth already described.

Single curve or involute gears will rum at varying distances between axes and transmit unvarying angular velocity.

This peculiarity makes these forms of gear teeth valuable for driving rolls or any rotating pieces, the distance between whose axes is likely to be changed.

A rack is a straight piece having teeth to mesh with a gear. A rack may be considered a gear of infinitely long radius. The circumference of a circle approaches a straight line as the radius inereases, and when the radius is infinitely long, any finite part of the circumference is a straight line. The pitch line of a rack, then, is a straight line tangent to the pitch circle of a gear meshing with the rack. All the dimensions of the parts of the teeth of a rack are calculated the same as for a wheel.

A rack to mesh with a single curve gear of 30 teeth or more is drawn as follows:

Draw straight pitch line of rack; also draw addendum line, working depth line and whole depth line, each parallel to the pitch line.

Point off the pitch line into parts equal to one-laalf the circular pitch, or $=t$.

Through these points draw lines at angles of $75 \frac{1}{2}^{\circ}$ with pitch lines, alternate lines slanting in opposite directions. These lines form the sides of rack teeth and are perpendicular to the lines of pressure.

In single curve gears of 12 and 13 teeth, the sides of spaces inside base circle, are parallel for a distance not more than $\frac{1}{3}$ of a diameter pitch, or $\frac{1}{3} s$; gears 14 , 15 and 16 teeth, not more than $\frac{1}{5} s ; 17$ to 20 teeth, not more than $\frac{1}{6} s$. In gears of more than 20 teeth the parallel construction is omitted.

To draw an involute gear of 12 teeth proceed as follows :

Draw pitch circle, base circle and addendum circle as in single curve gears of 30 teeth and over. Space off pitch line into parts equal to one-half circular pitch. Draw three or four tangents $i i^{\prime}, j j^{\prime}, k k^{\prime}, l l^{\prime}$ to the base circle, making the points of tangency, on
the base circle, $\frac{1}{4}$ of the circular pitch apart. With dividers set to $\frac{1}{4}$ the radius of pitch circle placing one leg in $i^{\prime}$ draw arc $x i j$; ( $x$ is on pitch circle) with leg in $j^{\prime}$ and radius $j j^{\prime}$ draw $j k$; with one $\operatorname{leg}$ in $k k^{\prime}$ and radius $k k^{\prime}$ draw $k l$. Should the addendum circle be outside $l$, complete tooth side with last radius.

These four ares together form a very close approximation to the true involute from the pitch circle. The exact involute for gear teeth is the curve made by the end of a band when unwound from a cylinder of the same diameter as the base circle.

Having obtained the involute for one side of the face of the tooth, reverse it for the other side.

From these two adjacent points on the base circle draw parallel lines to the centre of the gear. Then, with one leg of dividers in pitch circle in centre of next tooth, and the other leg just touching one of the parallel lines at $b$, continue the tooth side into $c$, where it meets a fillet are whose radius is $\frac{1}{6}$ the width of space at the addendum circle. This completes the flank of the tooth. This method is conventional-or not founded on principle other than the judgment of the designer. If flanks in any gear will clear addenda of a rack they will clear addenda of any other gear, not an internal gear. The addenda of teeth of this rack are rounded by a radius of $1 \frac{1}{4}$ pitch.

## Bevel Gear Blanks.

The pitch of bevel gears is always figured at the largest pitch diameter.

Most bevel gears connect shafts that are at right angles to each other. The following directions apply to any angle, but the sketch is made with axes at right angles.

Having decided upon the pitch, numbers of teeth and angle of shafts: (The sketch is made for gears of 4 pitch 12 and 24 teeth and three inches and six inches diameter.)

Draw axes $A O B$ and $C O D$. Draw line $m n$ parallel to $A O B$, bisected by $C O D$ and at a distance from $A O B$ equal to one-half the diameter of the large gear.

Draw the line $i j$ parallel to the line $C O D$, bisected by $A O B$ and at a distance from $C O D$ equal to onehalf the diameter of the small gear.

From ends of lines $m n, i j$ and at their intersection, draw lines to $O$.

These lines give size and shape of pitch cones. They are called Pitch Cone Lines.

Through points $m, i$ and $j$, draw lines $m x$, iy and $j$ z perpendicular to cone pitch lines.

On these lines, from cone pitch line, lay off distances for addenda, working depth and whole depth of teeth. From the points so obtained, draw lines to the centre $O$. These lines give the height of teeth above cone pitch lines, and the whole and working depths of teeth.

The teeth become smaller as they approach $O$ and become nothing at that point. It is quite as well never to have the length or face of teeth, $\mathrm{mm}^{\prime}$ longer than one-third the distance $O m$, nor more than two and a half times the circular pitch.

Having decided upon the length of face, draw limiting lines $m^{\prime} x^{\prime}, i^{\prime} j^{\prime}$ and $j^{\prime} z^{\prime}$.

We have now the outline of section of gears through their axes. A straight line drawn through the largest diameter of the teeth, perpendicular to axis of the gear is called the largest diameter. In practice these diameters are obtained by measuring the drawing. In this drawing the diameter of pinion is $3.45^{\prime \prime}$ and of gear 6.22".

To obtain data for teeth, we need only make drawing of section of one-lialf of each gear.

We first draw centre lines $A O, B O$ and the lines $g h$ and $c d$, then gear blank lines as in the case just described. (See sketch of S. C. Bevel Gear.)

To obtain shape of teeth in bevel gears, we do not lay them off on pitch circles in same way as in spur gears.

A line running from a point on cone pitch line to centre line of a bevel gear, perpendicular to this cone pitch line, is the radius for circle upon which to draw outlines of teeth at this point.

Hence $A c$ is the geometrical pitch circle radius, for large end of teeth, and $A^{\prime} c^{\prime}$ the geometrical pitch radius for small end of teeth of wheel. To avoid confusion, the distance $A^{\prime} c^{\prime}$ is transferred to $A c^{\prime \prime}$.

For the pinion we have the geometrical pitch circle radius $B c$ for large end of teeth, and the radius $B^{\prime} c^{\prime}$ for small end of teeth. Transfer distance $B^{\prime} c^{\prime}$ to line Bc.*

About $A$, draw arc crs and upon it lay off spaces equal to the thiçkness of tooth at pitch line, and draw outlines of teeth as previously described.

We have now the shape of teeth at large end, repeat this operation with radius $B c$ about $B$, and we have form of teeth, at large end of pinion.
*Tredgold's method from Rankine, App. Mech. p. 448.

Upon are of radius $A^{\prime} c^{\prime}$ we get shape of teeth of small end of gear, and upon arc of radius $B^{\prime} c^{\prime}$ we get shape of teeth at small end of pinion.

The sizes of tooth parts at small end may be taken directly from the diagram, or they may be calculated as follows :

Dividing the distance $O c^{\prime}$, which for example may be 2 inches, by $O c$, which may be 3 inches, we get $\frac{2}{3}$ or .666 for a ratio. Multiplying outside sizes by .666 , we get the correspouding inside sizes. Thickness of teeth at outside being . 314 inch, $\frac{2}{3}$ of it gives us .209 inch as thickness of teeth inside.

When cutting bevel gears with rotary cutters, the angle of cutter head is set the same as angle of working depth; thus: To cut the gear we have the cutter travel in the direction $O p$. The angle $A O_{p}$ is called the " cutting angle," being measured from the axis of the gear. In this method the angle of face of pinion is the same as cutting angle of gear, and face angle of gear is the cutting angle of pinion, and clearance is the same inside as outside.

## Epicycloidal Teeth.

An epicycloid is "a curved line generated by a point in the circumference of a circle, which rolls on the circumference of another circle, either internally or externally." (Worcester).

Hence an epicycloidal tooth has parts of epicycloids for the curves of its faces.

In the sketch, haring determined on one inch pitch, and found radius of rolling circle from diagram, the point $o$ was selected for a starting point, and the centre
of the rolling circle having its centre at $r^{\prime}$ rolled on the pitch circle until $r^{\prime}$ passed through a distance of $1 \frac{1}{8}$ inches, and the epicycloid traced by o found to be $o x$, for the face of the tooth, and in the same manner, the epicycloid $o x^{\prime}$ developed for the flank of the tooth. Four positions of the centre of the rolling circle, and four of the point $o$ are given in each case.

By the diagram thickness of tooth, $t=.48$ pitch. Length of tooth, $l=.7 P$. Three-tenths of this distance out from the pitch circle determines a point in the addendum circle, and four-tenths pitch in from the pitch circle gives a point in the whole depth circle. This allows $\frac{1}{10} P$ for clearance.

The curves obtained for one side of the tooth may now be reversed at a distance of $.48 P$, and we have the outline of a tooth that may be duplicated around the wheel.
" It is considered desirable by millwrights, with a view to the preservation of the uniformity of the shape of the teeth of a pair of wheels, that each tooth in one wheel should work with as many different teeth in the other wheel as possible.

They, therefore, study to make the numbers of teeth in each pair of wheels which work together, such as to be prime to each other, or to have their greatest common divisor as small as is possible consistently with the purposes of the machine.

The smallest number of teeth which it is practicable to give a pinion is regulated by the principle, that in order that the communication of motion from one wheel to another may be continuous, at least one pair of teeth should always be in action; and that in order to provide for the contingency of a tooth breaking, a second pair, at least shomld be in action also."

The least number of teeth that can nsually be employed is as follows:

Involute teeth, 25 ; epicycloidal teeth, 12 ; cylindrical teeth, or staves, 6.

The Arc of Contact on the pitch lines is the length of that portion of the pitch lines which passes the pitch point during the action of one pair of teeth; and in order that two pairs of teeth, at least may be in action at each instant, its length should be double the pitch. It is divided into two parts, the arc of approach and the arc of recess. In order that the teeth may be of length sufficient to give the required duration of contact, the distance moved over by the point $O$ on the pitch line, during the rolling of a rolling curve to describe the face and flank of a tooth, must be, in all, equal to the length of the required are of contact.

Line of Pressure. When one body presses against another, not attached to it, the tendency to move the second body is in the direction of the perpendicular at point of contact.

This perpendicular is called the line of pressure. The angle that this line makes with the path of the impelling piece is called the angle of pressure.

In the case of gearing the line of pressure makes an angle with the line of ceutres of $75^{\circ}$ to $78^{\circ}$.

## Proportions of Gear Wheels.

Much of the study and work on gears by engineers aud manufacturers has been devoted to the improvement of the shape of the teeth of gear wheels, and a large part of the illustrative chart is likewise occupied with representations of some of the most important of the results of this study and work. A matter often left to hap-hazard and "rule of thumb" design is the proportions of gear wheels. In the
sketch representing epicycloidal teeth, the design of a gear wheel is completed and the proportions so graphically represented as to enable a student with very small labor to design any gear. The proportions of a gear of less than 12 inch diameter are of comparatively small interest to the dranghtsman or designer since they have been so often designed and mannfactured that fairly perfect ones may be obtained of a dozen different manufacturers.

When, however, we need a gear wheel of from 2 to 10 feet diameter, we are, for various reasons, inclined to bestow considerable care on its design.

The proportions given in the sketch were compiled from the statements of three authors, and modified somewhat by personal experience. The pitch of the gear is here made the basis of all dimensions.

As to the relative strength of the different parts of a gear wheel, there is a wide difference of opinion; some holding that the teeth should be the weakest part and others contending that all parts should be equally strong-these latter having in mind the principle upon which the Deacon built his celebrated " one hoss shay."

## Teeth of Gear Wheels.

There are at least two good reasons why the teeth should be made the weakest part of a gear. The teeth are the smallest part of the gear, and in falling -after laving been broken-are least likely to damage the machine of which the gear is a part. Also, if but a few teeth or cogs are broken out, they may
be easily replaced by pins with small loss of timeafter which a new and better gear may be substituted.

The value of a tooth in transmitting power is a subject of great importance in this study of gears. The following is combined from Haswell, 'Tredgold and Rankine: At a speed of 400 feet per minute, a tooth whose breadth is to its thickness as 5 to 1 , and whose sectional area is one inch, will transmit six horse power.

The section of a one inch, circular pitch, cast gear of 2.25 inch face would give this section almost exactly.
2.25.


A cut gear of the same pitch would give a little larger section, but less strength, because it lacks the " skin" strength of the cast tooth.

A better proportioned tooth, for strength would be a 3 diametrical pitch $1 \frac{7}{8}$ inches wide. This pitch-cut
.53


$$
1.875
$$

gear-gives us an area of ( $1,875 \mathrm{in} . \mathrm{x} .53 \mathrm{in}$.) . 01 of a square inch (.00875) less than the one just mentioned, but its strength is more than $7 \%$ stronger ( $b d^{2}$ ) than the other.

A tooth, then, of 3 diameter pitch, $1 \frac{7}{8}$ inch face, moving at a velocity of 400 feet per minute, will transmit 6 HP .

There seems to be an ample factor of safety in this case, and if this rule were followed there would rarely be a break under the strain for which the gear was desigued.

We must not, however, forget irregular motions caused by varying power and varying work, blowholes and slag weakening the iron, backlash and falling dirt.

It must be remembered that gears should be so designed that two teeth of each gear are always in mesh. This sometimes prevents the loss of more than one tooth, from a defect in the iron and practically reduces the strain on the tooth one-half. This tooth we have just arrived at is larger than is needed for fine gearing, where poor iron, irregular motion and backlash are reduced to a minimum, but we are considering cast, not cut gears.

The figures at the left hand, of the right hand diagram, in the left hand npper corner of the sketch, represent "the number of pounds strain each pitch will safely transmit per inch width of wheel face "according to Prof. Marks. Probably Prof. Marks considered the tooth as a beam built in at one end, with the load at the other, which is an extreme case. Also the element of motion is not taken into account.

## GEAR DESIGN.

After the cross-sectional area of the tooth is determined on, the proportions for the rest of the gear are easily arrived at by looking at the sketch and accompanying diagram.

In the diagram, the vertical column of figures indicate circular pitch.





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The horizontal lines are simply divisions of the pitch-or pitches.

The inclined lines show the variations of wheel dimensions corresponding to the pitch. Thus, small $r$, the radins of rolling eircle, for a 4 inch pitch gear is $\frac{7}{8}$ of 4 inches, or $3 \frac{1}{2}$ inches, for 3 inch pitch, $r$ is $2 \frac{5}{8}$ inches, for 2 inch pitch, $r$ is $1 \frac{3}{4}$ inches, and for 1 inch piteh $r$ is $\frac{7}{8}$ of an inch.

This matter of the size of the rolling eircle is a very important one. Its size may be increased until the flanks of the teeth are straight radial lines, or decreased until the face of the tooth is an are of as small a circle as the rolling circle itself, $i, e$, an epicycloid which nearly coincides with an are of a circle equal to the rolling circle.

Looking now for the thickness of rim, we find that it is given in the diagram as $d=\frac{1}{8}$ inch $+.4 P$, and this for 4 inches $P$ is 1.735 inches, or about $1 \frac{3}{4}$ inches, for 3 inches pitch $d$ is $1 \frac{11}{32}$, for 2 inches $\frac{15}{16}$ of an inch, and $\frac{17}{32}$ of an inch for 1 inch pitch. This rim is increased from the edge or side of the wheel, toward the centre, until it is 1.2 d thiek, where it is reinforced by a central rib $d$ wide and $d$ thick. This rim has had its teeth stripped from it, and therefore is strong enough, though it looks light.

For computing the strength of arms we must have the pitch diameter and width of face of wheel given. Denoting the face of the wheel by $b$, half the pitch diameter by $R$, and the required depth of the arm at the hub by $h$, the following formule for arms are considered good:

$$
\begin{aligned}
& \text { For } 4 \text { arms } h=.61 V_{b r} \\
& \text { For } 6 \text { arms } h=.5 V_{b r .} \\
& \text { For } 8 \text { arms } h=.46 \text { Vbr. } \\
& \text { For } 10 \text { arms } h=.443 \text { Vr. } \\
& \text { For } 12 \text { arms } h=.438 \text { Vr. }
\end{aligned}
$$

The depth of the arm at the rim should be $\frac{2}{3}$ of the hub depth.

The term depth is here used to denote the dimension of the arm in a plane at right angles to the axis of rotation.

To strengthen the arm against side thrust and tw'sting, a rib $a$ is put on each side of the arm, nearly as wide as the face of the gear. Its thickuess is .7 d .

The fillet between the arms at the hub shotald never be less than $\frac{1}{2} d$ deep.

The thickness of the hub is made $4 \frac{3}{16}$ inches for a wheel on a 10 inch shaft, and $\frac{3}{10}$ of an inch less for each inch decrease of diameter of the shaft. An increase in these sizes for all shafts over 6 inches in diameter would be an improvement.

With this explanation of the graphical representation of a gear design before you, you have sufficient direction to design gear wheels.

It may be well to add here that no amount of "book wisdom" will supply a want of practical knowledge of the subject in hand, nor will any amount of "finger wisdom" enable a mechanic to design the gear he fanltlessly makes.

Where there is a call for a special gear, it must be adapted to special conditions. In such a case, the man of most experience, available, is called in and his design relied on. Experience, judgment and "common sense," as it is called-though it is rather uncom-mon-are necessities to the designer. These coupled with "book wisdom" ought to make a good designer. If in addition to these qualities he has plenty of "finger wisdom" and a natural mechanical ability his designs should be as nearly perfect as our civilization demands.

Such a designer as this will never fall into the habit of his lesser brethren, of " putting in a little more iron" when the exigencies of the case are not easily arrived at.

There are, to be sure, many cases where it is plain
that a few cents' worth more of iron in a casting may save a very expensive break, and few indeed there are who hesitate to add the few cents' worth of iron.

To closely calculate the amount and direction of stress that comes upon a gear in a peculiar position and successfully design the gear for such position, is the work of a practical designer. The term " practical" here used only attains its full force when the gear is designed speedily.

## Strength of Materials.

For the student's convenience, a few tables and alphabets are here provided.

Strength is the resistance a body opposes to a permanent separation of its component parts. Elasticity is the resistance a body opposes to a change of form. The three terms tensile, crushing and transverse, as used in the tables, are readily understood.

A study of the tables will give the student an accurate standard of the comparative strength of materials.

It will be noted, for example, that cast iron is better fitted for posts than wrought iron, while the latter is superior to the former for beams or rods. In this commection it will be noted also that when the natural order of things is reversed there should be an equivalent change of form and size of the material. There are a multitude of facts bearing on the use of materials, but there is room here for only one more.

The strength of a beam of any material, increases directly as to the breadth, and as the square of the depth. This fact is easily remembered in the formula bdz. Hence the transverse strength of a 2 by 6 beam placed edge up is greater than that of a 3 by 6 beam placed side $u p$, in the ratio 72 to 54 .

Strength ny moterials.


| Areas of Circles. |  |  |  |  |  |
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| Dram | rea |  |  |  | ares |
| $\frac{1}{8}$ | . 012272 | 21 | 346.361 | 48 | 180\%156 |
| $\frac{1}{4}$ | .049087 | 22 | 380.134 | 49 | 1885,95 |
| $\frac{3}{8}$ | . 110447 | 23 | 415.477 | 50 | $1963 .{ }^{2}$ |
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| $\frac{5}{8}$ | . 306796 | 25 | 490.875 | U2 | 2123 72 |
| $\frac{4}{4}$ | . 441787 | 26 | 530.93 | 53 | 2206.19 |
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| 4 | 12.5664 | 31 | 754.769 | O8 | 2642.09 |
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| 6 | 28.2444 | 3.3 | 855.301 | 60 | 2897.44 |
| 7 | 38.48'46 | 34 | 907.922 | C/ | 292247 |
| 8 | 002606 | 30 | 962.115 | 62 | 3019.08 |
| 9 | 63.6174 | 36 | 1017.878 | $60^{3}$ | 3117.20 |
| 10 | 78.54 | 37 | 1075.213 | 64 | 32/7. |
| $1 /$ | 95.0334 | 38 | 1/34.118 | $60^{\circ}$ | $33 / 8.31$ |
| 12 | 113.098 | 39 | 1184.59 .3 | 66 | 3421.2 |
| 13 | 132.733 | 40 | 1256.64 | 67 | 3525.66 |
| 14 | 153.938 | 41 | 132026 | 68 | 3631.69 |
| 15 | $1 \pi 6.715$ | 42 | 1385.45 | 69 | 373929 |
| 16 | 201.062 | 43 | 1452.2 | 70 | 384846 |
| 17 | 226.981 | 44 | 1520.53 | 71 | 37592 |
| 18 | 254.47 | 45 | $1570.43^{3}$ | 72 | 40\%10\% |
| 19 | 283.529 | 46 | 1661.91 | 73 | 4180.4 |
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| $\frac{1}{2}$ | 15708 | 24 | 75.3984 | 51 | 160.222 |
| $\frac{5}{8}$ | 1.9635 | 25 | 78.54 | 52 | 163363 |
| $\frac{3}{4}$ | 2.3562 | 26 | 81.6816 | 53 | 166.505 |
| $\frac{8}{8}$ | 27489 | 27. | 848232 | 54 | 169.646 |
| 1 | 3.1716 | 26 | 87.9648 | 55 | 172.788 |
| 2 | 62832 | 29 | 911064 | 56 | 175.93 |
| 3 | 94248 | 30 | 94.248 | 57 | 179.071 |
| 4 | 12.5664 | 31 | 97.3896 | 58 | 182.2/3 |
| $\hat{6}$ | 15706 | 32 | 100.531 | 59 | 185.354 |
| 6 | 18.8496 | 33 | 103.673 | 60 | 188.496 |
| $\gamma$ | 2199 | 34 | 106.814 | 61 | 191.638 |
| $\delta$ | 25132.8 | 35 | 109.956 | 62 | 194.779 |
| 9 | 28.2744 | 36 | /13.098 | 63 | 197.921 |
| 10 | 31416 | 37 | 116.239 | 64 | 201.062 |
| $1 /$ | 345576 | 38 | 1119.381 | 65 | 204.204 |
| 12 | 371992 | 39 | 122.522 | 66 | 207.346 |
| 13 | 408408 | 40 | 125664 | 67 | 210.487 |
| 14 | 43.9824 | 41 | $128^{\prime} 806$ | 68 | 213.629 |
| 15 | 47124 | 42. | 131947 | 69 | 216.77 |
| 16 | 502656 | 43 | 135.087 | 70. | 219.912 |
| 17 | 534072 | 44 | 138.23 | 71 | 223054 |
| 18 | 56.5188 | 45 | 141.372 | 72 | 226.195 |
| 19 | 59.69 | 46 | 144.514 | 73 | 229.337 |
| 20 | 62.832 | 47 | 147655 | 24 | 232.478 |


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|  | 8 ths | 16 ths. | 32 nd | 6aths | 64ths |
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| $0^{3}$ | , 375 | . 187 | . 093 | . 046 | . 546 [35 |
| 5 | . 625 | . 312 | . 156 | . 078 | . 5788.37 |
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| 9 |  | . 562 | 281 | . 140 | . 64041 |
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| 13 |  | . 812 | 406 | 203 | .703 45 |
| 5 |  | .937 | . 468 | . 234 | 73447 |
| 17 |  |  | . 531 | . 265 | 760549 |
| 19 |  |  | . 59.3 | . 296 | . 79651 |
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| $23^{3}$ |  |  | . 718 | . 359 | . 85955 |
| 25 |  |  | . 781 | . 390 | . 89057 |
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## ECYPTIAN

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